Problem 15

A driver sets out on a journey. For the first half of the distance she drives at the leisurely pace of 30 mi/h; she drives the second half at 60 mi/h. What is her average speed on this trip?

Solution

Let the total distance of the trip be d, and calculate the average speed from the defining formula.

$$\begin{aligned}
\bar{s} &= \frac{\displaystyle\int s(x)\,dx}{\displaystyle\int dx} \\
&= \frac{\displaystyle\int_0^{d/2} 30\,dx + \int_{d/2}^{d} 60\,dx}{\displaystyle\int_0^{d} dx} \\
&= \frac{\displaystyle 30\int_0^{d/2} dx + 60\int_{d/2}^{d} dx}{\displaystyle\int_0^{d} dx} \\
&= \frac{30\left(\frac{d}{2}-0\right)+60\left(d-\frac{d}{2}\right)}{d-0} \\
&= \frac{30\left(\frac{d}{2}\right)+60\left(\frac{d}{2}\right)}{d} \\
&= \frac{15d+30d}{d} \\
&= \frac{45d}{d} \\
&= 45
\end{aligned}$$

Therefore, the driver's average speed on this trip is 45 miles per hour.